

Incomplete Market Equilibrium under Social Comparisons

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- **Peer effects on consumption are empirically well-documented:**
 - Consumption decisions are influenced by one's peers (Kuhn et al., 2011; Agarwal et al., 2021).
 - Households perceiving themselves as poorer save less (Georgarakos et al., 2014; Bertrand and Morse, 2016)
- **Changes in consumption propagate through the social network:**
 - *Feedback loop*: changes in my consumption affect my peers' consumption, which feeds back to me (Langtry, 2023)
- **In a Lucas economy, aggregate consumption is exogenously fixed:**
 - Social comparisons cannot amplify aggregate quantities
 - ⇒ *reallocation* of consumption
 - ⇒ changes in demand for financial assets
 - ⇒ **changes in asset prices**
- **Social comparisons interact asymmetrically with risk:**
 - *Hedgeable risk vs Unhedgeable income risk*

Two assets:

- risk-free asset in *zero* net supply, paying interest rate r_t ,
- stock in *unit* net supply, paying a dividend stream D_t .

Dividend dynamics:

$$dD_t = \mu_D dt + \sigma_D dW_t^D.$$

The interest rate r_t and the stock price S_t are determined *endogenously* in equilibrium.

Continuum of agents:

Agents are indexed by $h \in \mathcal{H}$, with population measure $m(\cdot)$ on $\mathcal{H}$. Each agent h is characterized by:

- preferences: risk tolerance $\tau^h > 0$, discount rate $\beta^h > 0$
- initial stock endowment v^h ; income process (b^h, σ^h, ρ^h)
- comparison weights

Income dynamics:

$$dg_t^h = b^h dt + \sigma^h(\rho^h dW_t^D + \sqrt{1 - (\rho^h)^2} dW_t^h).$$

- $\rho^h dW_t^D$: correlated with dividends $\rightarrow$ *hedgeable*
- $\sqrt{1 - (\rho^h)^2} dW_t^h$: idiosyncratic, independent across agents $\rightarrow$ *unhedgeable*

$\Rightarrow$ Markets are **incomplete**: agents cannot fully insure their labor income risk

Utility maximization:

Agent h maximizes lifetime expected utility from consumption *relative to social benchmark* A^h :

$$\max_{(c, \theta) \in \mathcal{A}} E \left[\int_0^\infty e^{-\beta^h s} U(c_s; \tau^h, A_s^h) ds \right],$$

where

$$U(c; \tau, A) = -\tau \exp \left\{ -\frac{1}{\tau} (c - A) \right\}.$$

$(A_s)_{s \geq 0}$ aggregates consumption across the population according to each agent's kernel $\pi(\cdot, \cdot)$.

- i) Each agent takes A^h as given and optimizes.
- ii) In equilibrium, benchmarks are consistent with consumption policies:

$$A_t^h = \int_{\mathcal{H}} \pi(h, h') E[c_t^{h'} | \mathcal{F}_t^D] dm(h').$$

$$\pi(h, h') = \sum_{i=1}^R \phi_i(h) \psi_i(h')$$

- Finite rank approximation of general kernel $\pi(h, h')$
- *Economic interpretation:*
Each index $i \in \{1, \dots, R\}$ represents a social feature
 - $\psi_i(h')$: agent h' 's score on feature i (income, education, neighborhood, ...)
 - $\phi_i(h)$: how much agent h cares about feature- i consumption

Special case: cluster network

Partition $\mathcal{H}$ into R groups:

$$\psi_i(h) = \mathbf{1}_{h \in \mathcal{C}_i}, \quad \phi_j(h) = G_{ij} \quad \text{for } h \in \mathcal{C}_i$$

G_{ij} : weight cluster i places on cluster j 's consumption

A **mean-field market equilibrium** is a collection of prices (r_t, S_t) , policies $(c_t^h, \theta_t^h)_{h \in \mathcal{H}}$, and reference points $(A_t^h)_{h \in \mathcal{H}}$ satisfying:

1. each agent h maximizes expected utility, taking A^h as given;
2. each agent h 's consumption is financed by their trading strategy θ^h ;
3. markets clear:

$$\begin{aligned} \int_{\mathcal{H}} E[\theta_t^h | \mathcal{F}_t^D] dm(h) &= 1, \quad \int_{\mathcal{H}} E[X_t^h - \theta_t^h S_t | \mathcal{F}_t^D] dm(h) = 0, \\ \int_{\mathcal{H}} E[c_t^h | \mathcal{F}_t^D] dm(h) &= D_t + \int_{\mathcal{H}} E[g_t^h | \mathcal{F}_t^D] dm(h) \quad \text{a.s. for all } t > 0; \end{aligned}$$

4. the consistency condition holds:

$$A_t^h = \int_{\mathcal{H}} \pi(h, h') E[c_t^{h'} | \mathcal{F}_t^D] dm(h').$$

Solution: Optimal Policies

Write $E[f] := \int_{\mathcal{H}} f(h) dm(h)$ for population averages.

Theorem

Define $\Phi := [\phi_i]_{i=1}^R$, $\Psi := [\psi_i]_{i=1}^R$.

Equilibrium Portfolio:

$$\theta = \frac{\tau}{\bar{\tau}} - \frac{\sigma\rho - \frac{\tau}{\bar{\tau}}E[\sigma\rho]}{\sigma_D} + \frac{\sigma_D + E[\sigma\rho]}{\bar{\tau}\sigma_D} \left(\Phi - \frac{\tau}{\bar{\tau}}E[\Phi] \right)^\top \mathcal{M} E[\tau\Psi]$$

where

$$\bar{\tau} := E[\tau], \quad \mathcal{M} := \left(I - E \left[\left(\Psi - \frac{1}{\bar{\tau}}E[\tau\Psi] \right) \Phi^\top \right] \right)^{-1}.$$

Equilibrium Consumption c_t has an explicit expression of the form:

$$c_t = g_t + r(x_0 - \theta S_0) + \theta D_t + \Lambda \cdot \left(t - \frac{1}{r} \right),$$

where Λ captures individual preferences, income parameters, and social comparison corrections.

Both θ and c_t depend on the agent's *position in the network* through $\mathcal{M}$.

Social Multiplier Matrix

$$\mathcal{M} := \sum_{k \geq 0} \left(E \left[\left(\Psi - \frac{1}{\bar{\tau}} E[\tau \Psi] \right) \Phi^\top \right] \right)^k$$

- Total W^D -volatility in the economy is fixed at $\sigma_D + E[\sigma \rho]$
- **Without social comparisons:** agent's W^D -risk exposure is

$$V_D^{(0)} = \frac{\tau}{\bar{\tau}} (\sigma_D + E[\sigma \rho])$$

- **With social comparisons:** agent wants to add the W^D -volatility of benchmark A_t
 $\hookrightarrow$ aggregate exposure is fixed, so desired increases must be offset across agents:

$$V_D^{(i+1)} = \frac{\tau}{\bar{\tau}} (\sigma_D + E[\sigma \rho]) + \underbrace{\Phi^\top E[\Psi V_D^{(i)}]}_{\substack{W^D\text{-exposure} \\ \text{of benchmark } A_t}} - \frac{\tau}{\bar{\tau}} E \left[\Phi^\top E[\Psi V_D^{(i)}] \right].$$

$\hookrightarrow$ iterating generates successive rounds of risk reallocation through the network

Fixed point of the recursion yields $\mathcal{M}$ and the equilibrium risk exposure:

$$V_D = \frac{\tau}{\bar{\tau}} (\sigma_D + E[\sigma\rho]) + \left(\Phi^\top - \frac{\tau}{\bar{\tau}} E[\Phi]^\top \right) \mathcal{M} E\left[\frac{\tau}{\bar{\tau}} \Psi\right] (\sigma_D + E[\sigma\rho])$$

↪ achieved by holding θ shares of the stock: $\sigma\rho + \theta\sigma_D = V_D$

Cluster Network

R clusters of equal mass $p = 1/R$.

- Matrix $E[\Psi\Phi^\top]$ has entry $p G_{ij}$
↪ captures *network amplification* as in Bonacich centrality
- Matrix $E[(\Psi - \frac{1}{\bar{\tau}} E[\tau\Psi])\Phi^\top]$ has entry $p\left(G_{ij} - \frac{\tau_i}{\bar{\tau}} \frac{1}{R} \sum_{l=1}^R G_{lj}\right)$
↪ what matters is cluster i 's sensitivity to cluster j *relative to the average*
↪ in aggregate: no amplification, only *reallocation* between clusters

Theorem

Equilibrium Interest Rate:

$$r = \frac{1}{\tau_{MF}} \left(E[\tau\beta] + \mu_D + E[b] - E \left[\frac{\sigma^2(1 - \rho^2)}{2\tau} \right] \right) - \frac{1}{2\tau_{MF}^2} (\sigma_D + E[\sigma\rho])^2 \\ + \frac{1}{\bar{\tau}} E[\Phi]^T \mathcal{M} E \left[\left(\tau\beta - \frac{\sigma^2(1 - \rho^2)}{2\tau} \right) \Psi \right],$$

where

$$\tau_{MF} := \bar{\tau} \left(1 - \frac{1}{\bar{\tau}} E[\Phi]^T \mathcal{M} E[\tau\Psi] \right)^{-1}.$$

Equilibrium Stock Price:

$$S_t = \frac{D_t}{r} + \frac{\mu_D}{r^2} - \frac{1}{\tau_{MF} r^2} \sigma_D (\sigma_D + E[\sigma\rho]).$$

In the absence of social comparisons ($\Phi = 0$), the model reduces to Christensen et al. (2014).

When comparisons raise risk tolerance

With homogeneous discount rates and no uninsurable income, asset prices same as in a representative-agent economy with risk tolerance τ_{MF} .

Proposition

Suppose $\Phi, \Psi \geq 0$ componentwise and $\text{spr}(E[\Psi\Phi^\top]) < 1$. Then

$$\tau_{MF} \geq \bar{\tau}.$$

- **Mechanism:**

social benchmark co-moves with individual consumption

⇒ lower utility cost of bearing market risk

⇒ agents require less compensation (stronger demand for the risky asset)

⇒ lower market price of risk:

$$\frac{1}{\tau_{MF}}(\sigma_D + E[\sigma\rho])$$

- **Network structure matters:**

τ_{MF} is largest when the most risk-tolerant agents are *central*

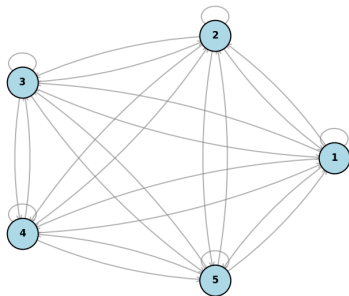
Network Structure

The population is divided into R clusters of equal mass $1/R$.

$G = \lambda W$ is the comparison matrix,

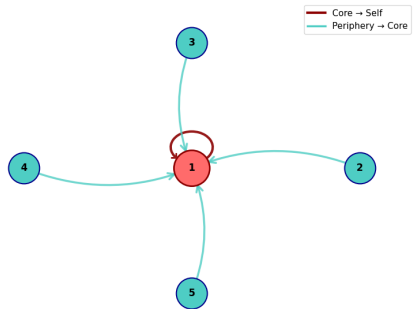
$\lambda \geq 0$ is comparison intensity and W is row-stochastic: $W_{ij} \geq 0$, $\sum_{j=1}^R W_{ij} = 1$.

Fully Connected Network



$$W_{ij} = \frac{1}{5} \text{ for all } i, j$$

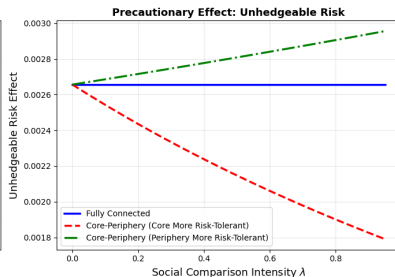
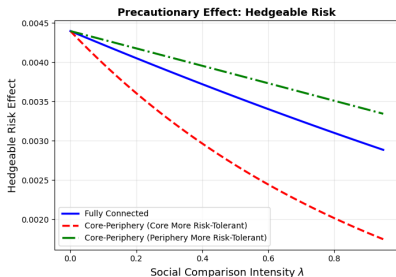
Core-Periphery Network



$$\text{Core: } W_{11} = 1$$

$$\text{Periphery: } W_{ii} = 1 \text{ for } i \in \{2, 3, 4, 5\}$$

Precautionary Savings



- Dividend risk: comparisons reduce precautionary effect.
- Idiosyncratic income risk: effect depends on network structure.
- If agents with *low* precautionary motives ($\sigma^2(1 - \rho^2)/2\tau$ small) are central:
 - ↪ their high consumption raises others' benchmarks
 - ⇒ precautionary savings *reduced*
- If agents with *high* precautionary motives are central:
 - ↪ elevated precautionary savings propagate through network
 - ⇒ effect *amplified*

- **Framework:**

- we introduce the notion of *mean-field market equilibrium*: market clearing + MF consistency condition
- derive closed-form solution via social multiplier matrix $\mathcal{M}$

- **Findings**

- *Reallocation, not amplification:*
 $\mathcal{M}$ captures how network interactions *redistribute* consumption and risk across agents, with equilibrium market-clearing as the binding constraint
- *Higher effective risk tolerance:*
social comparisons lower the market price of risk
- *Asymmetric treatment of risk:*
for unhedgeable income risk, the precautionary savings effect depends on *who is central* in the network
- MF market equilibrium is the $n \rightarrow \infty$ limit of n -agent market equilibria
Idiosyncratic shocks average out: tractable and interpretable model

Thank you!

Questions or comments?

Each agent has size $1/n$:

$$\underbrace{\gamma^i}_{\text{stock endowment}} = \frac{v^i}{n},$$

$$\underbrace{g_t^i}_{\text{labor income}} = \frac{g_t^i}{n},$$

$$\underbrace{\delta^i}_{\text{risk tolerance}} = \frac{\tau^i}{n}.$$

Convergence to the mean-field equilibrium

As the empirical type distribution converges to m :

- **Asset prices:** $r^{(n)} \rightarrow r$ and $S^{(n)} \rightarrow S$ almost surely, uniformly on finite time intervals.
- **Individual agents:** Consumption, stock holdings and social benchmarks, multiplied by n , approach those of mean-field agents with the same parameters and Brownian shocks.

When topology drops out of prices

Two sufficient cases

- **Identical agents:** all clusters have the same preference and income parameters. Any row-stochastic W gives the same prices.
- **Balanced influence:** W is doubly stochastic. Prices are independent of its topology even with heterogeneous agents.

- **Agent h 's controls:**

- stock holdings θ_t^h ,
- consumption rate c_t^h .

- **Wealth X_t^h dynamics:**

$$dX_t^h = \theta_t^h(dS_t + D_t dt) + r_t(X_t^h - \theta_t^h S_t) dt + g_t^h dt - c_t^h dt$$

- Wealth changes from:

- capital gains and dividends $\theta_t^h(dS_t + D_t dt)$
- interest income on risk-free position $r_t(X_t^h - \theta_t^h S_t) dt$
- labor income $g_t^h dt$
- consumption $c_t^h dt$

- Recall: (r_t, S_t) are determined endogenously in equilibrium

Finite-agent convergence

Rank-one kernel: $\pi(h, h') = \phi^h \psi^{h'}$ with nonnegative factors.

- Type parameters lie in a common compact set, with $\tau \geq \tau_* > 0$, and their empirical distribution converges to the mean-field type distribution.
- The limiting economy satisfies the equilibrium assumptions and $r > 0$.

For every finite T , prices converge almost surely:

$$|r^{(n)} - r| + \sup_{0 \leq t \leq T} |S_t^{(n)} - S_t| \longrightarrow 0.$$

Match agent i to a mean-field agent with the *same parameters and Brownian shocks*, denoted $(\bar{\theta}^i, \bar{c}^i, \bar{A}^i)$. Then, almost surely,

$$\begin{aligned} \max_{i \leq n} \Big(& |\theta^{i,(n)} - \bar{\theta}^i| + \sup_{0 \leq t \leq T} |c_t^{i,(n)} - \bar{c}_t^i| \\ & + \sup_{0 \leq t \leq T} |A_t^{i,(n)} - \bar{A}_t^i| \Big) \longrightarrow 0. \end{aligned}$$

Precautionary Savings

Social comparisons interact *asymmetrically* with hedgeable and unhedgeable risk:

$$r = (\dots) - \underbrace{\frac{(\sigma_D + E[\sigma\rho])^2}{2\tau_{MF}^2}}_{\text{hedgeable: dampened}} - \underbrace{\frac{1}{\tau_{MF}} E\left[\frac{\sigma^2(1-\rho^2)}{2\tau}\right] - \frac{1}{\bar{\tau}} E[\Phi]^\top \mathcal{M} E\left[\frac{\sigma^2(1-\rho^2)}{2\tau} \psi\right]}_{\text{unhedgeable: sign depends on network}}$$

- **Hedgeable dividend risk:**

co-movement between consumption and benchmark *dampens* precautionary savings

- **Unhedgeable income risk:**

idiosyncratic shocks are independent across agents

↪ effect on precautionary savings depends on network structure

How the equilibrium is constructed

1. Fix affine reference processes

$$A_t^h = \gamma_0^h + \gamma_1^h t + \gamma_2^h W_t^D.$$

Discounted marginal utility gives an agent-specific stochastic discount factor:

$$y^h M_t^h = \exp \left\{ -\beta^h t - \frac{c_t^h - A_t^h}{\tau^h} \right\}.$$

2. Use financing and market clearing

- The financed net-consumption claim must be replicable by the two traded assets.
- Market clearing identifies the interest rate and the common price of dividend risk.

3. Impose benchmark consistency

$$z_t := \int_{\mathcal{H}} \Psi(h) E[c_t^h \mid \mathcal{F}_t^D] dm(h), \quad A_t^h = \Phi(h)^\top z_t.$$

Finite rank reduces the benchmark coefficients to linear systems of dimension R .