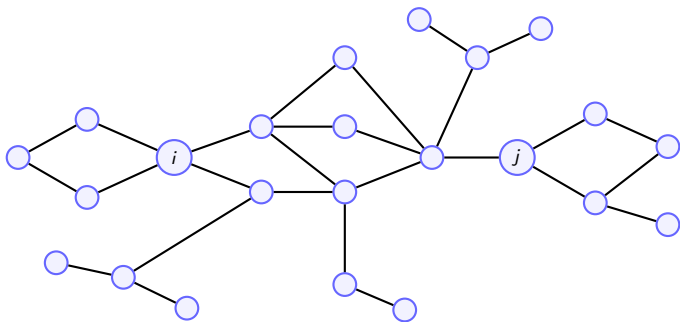


Stable Governments and Coups in Political Networks



Matthew O. Jackson
Stanford University

Shahir Safi
New Economic School

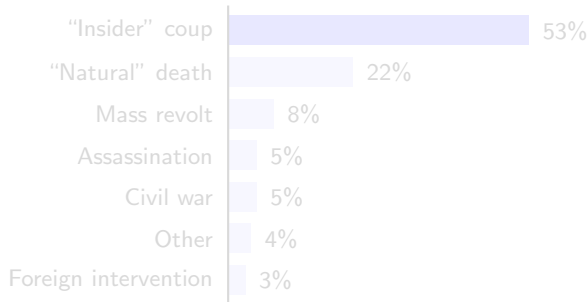
Konstantin Sonin
University of Chicago

How Autocrats Lose Power

663 autocrats, 1946-2008 (data from Svolik 2012).

246 left through term limits or elections.

The other 404:



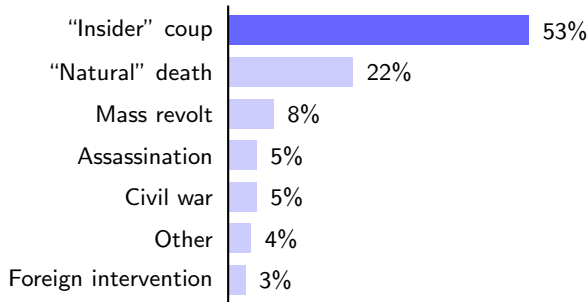
Most removals come from inside the ruling elite.

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Most removals come from inside the ruling elite.

Coups

- 1964: Khrushchev, by Brezhnev, Kosygin, Suslov (Politburo, Central Comm.)
- 1973: Salvador Allende, by General Augusto Pinochet
- 1799: the Directorate, by Napoleon Bonaparte
- 1922: Luigi Facta (under Victor Emmanuel III), by Benito Mussolini
- 1936, 1981: Spain
- ...

Stable Political Structures

- Overthrowing leadership requires a motivated mutiny
- Network positions influence who prefers whom, who joins
- Which network structures allow a leader's survival?

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- CEOs of organizations
- Citizen candidates
- Location choices

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Road Map

1 Model

2 Existence

- Centrality: Trees, Median and Modular Networks
- Balance: Core-Periphery Networks

3 Robustness

4 Conclusion

A Model

- $N = \{1, \dots, n\}$: agents.
- Undirected network g , where agents are nodes and links are political ties.
- $d(i, j)$: distance (length of shortest path) between i and j .
- Agents prefer leaders closer to them. Supporters of j against i :

$$P(j, i) = \{k : d(k, j) < d(k, i)\}.$$

Equidistant agents abstain.

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Coup Stability

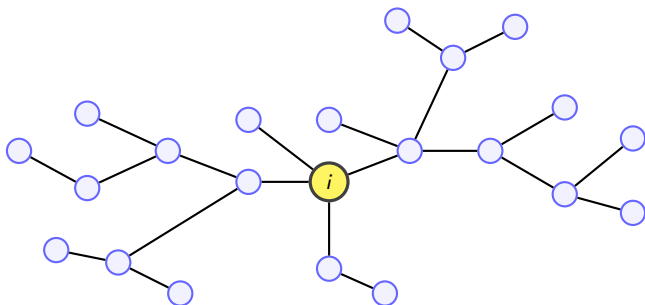
Node i is **coup stable** if there does not exist any j such that

$$|P(j, i)| > \frac{n}{2}$$

or equivalently

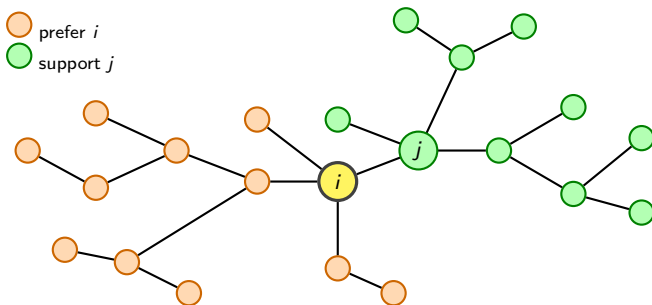
$$|\{k \in N : d(k, j) < d(k, i)\}| > \frac{n}{2}.$$

Start on a Tree



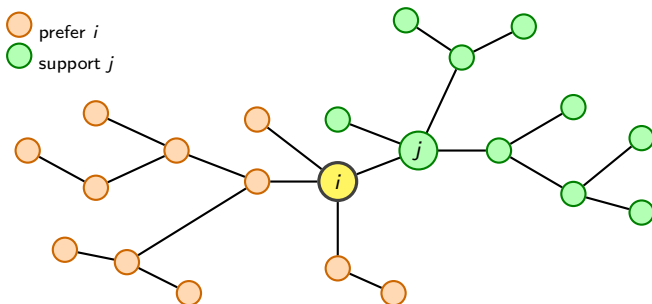
- $n = 22$. Node i is the median: $i \in \arg \min_m \sum_h d(m, h)$.
- Challenger j , a neighbor of i : only its branch prefers j , $10 < 11$. Everyone else reaches j **through** i .
- Every branch has at most $n/2 = 11$ nodes, and a challenger deeper in a branch attracts even fewer. So i is coup stable.

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From the Literature

- Demange (1984): single-peaked preferences on trees $\Rightarrow$ Condorcet winners.
- Barthélemy and Monjardet (1981), Bandelt and Barthélemy (1984), Buechel (2014): cube-free median networks admit Condorcet winners.

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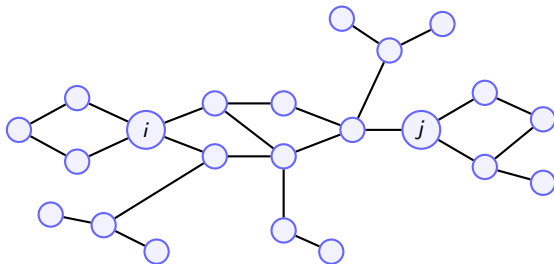
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Median Networks

- $I(i, j)$: nodes on shortest paths between i and j , endpoints included.
- Median network: every triple has a unique intersecting node,

$$|I(i, j) \cap I(j, k) \cap I(i, k)| = 1 \text{ for all distinct } i, j, k.$$

- Trees are median networks, but median networks can have cycles.



Nonexistence

Not a median network: hubs 1, 2, 3 have no intersecting node.

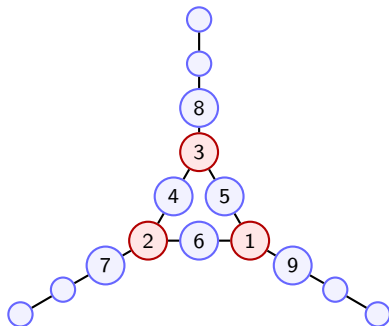
Three equal factions, led by hubs 1, 2, 3, joined on a six-cycle.

Hub 1 loses to 4: node 4 sits between the other two factions and unites them, 9 votes to 6.

But 4 loses to hub 3, which unites its own faction with 1's, 9 votes to 6.

The cycle continues:

$1 \rightarrow 4 \rightarrow 3 \rightarrow 6 \rightarrow 2 \rightarrow 5 \rightarrow 1$ (each beaten by the next). **No coup-stable node.**



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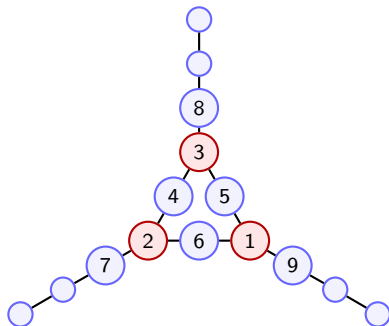
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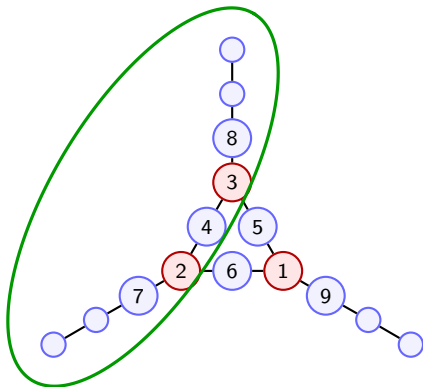
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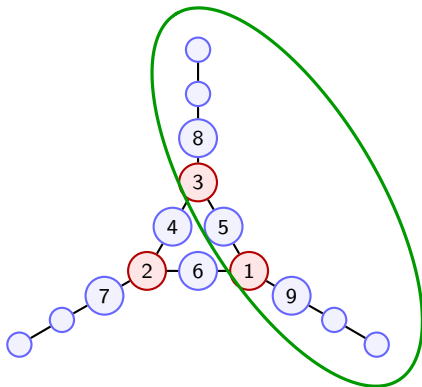
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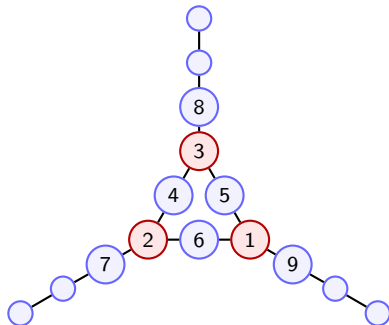
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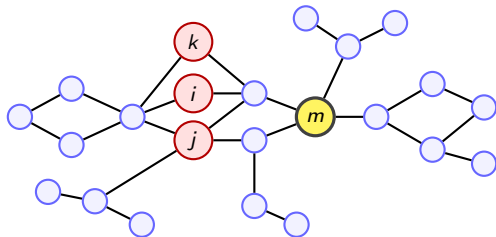
- Which other classes of networks have coup-stable nodes?

Modular Networks

- Modular network: every triple has **at least one** intersecting node,

$$|I(i,j) \cap I(j,k) \cap I(i,k)| \geq 1 \text{ for all distinct } i,j,k.$$

- Trees and median networks are modular.



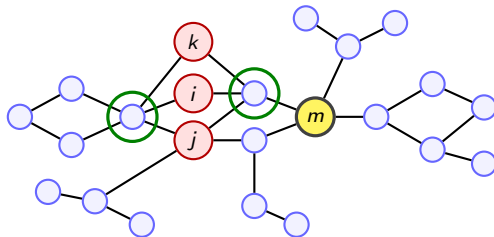
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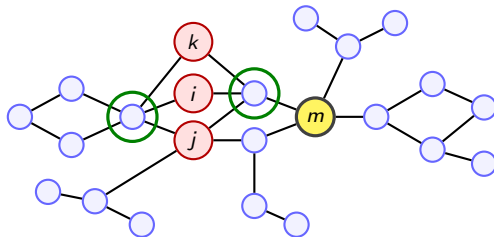
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Consider a modular network with no chordless cycles of six or more nodes. Then every median node is coup stable.

- Covers trees and median networks without large chordless cycles.
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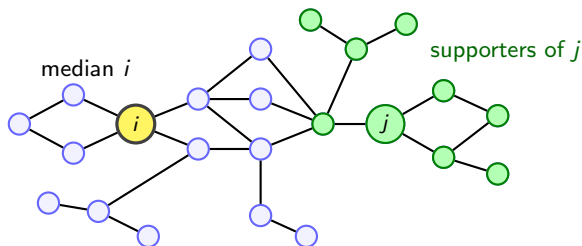
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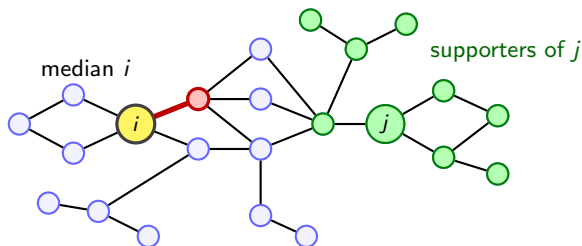
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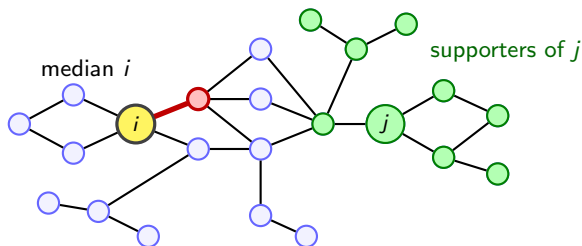
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- Move from i one step closer to j 's supporters: a strict majority gets closer, everyone else is at most one step farther.
- Total distance falls, contradicting i being a median.
- **The crux:** finding one step that brings every supporter of j closer, a neighbor linked to the whole next layer toward j . This is where no long chordless cycles is needed.

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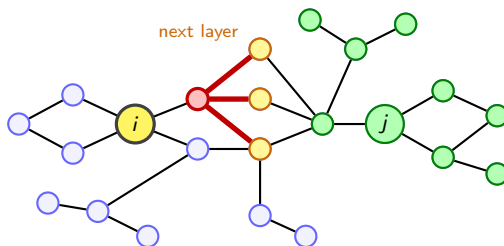
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Two Logics of Coup Stability

- **Centrality:** in hierarchies, a highly central node beats, or does not lose to, every other node.
- **Balance:** in core-periphery networks, a group of roughly balanced nodes, none of which beats another.

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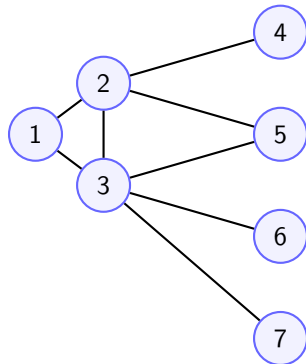
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Split Networks

Networks with two sets of nodes:

- a clique (the core), and
- an independent set (the periphery).

Core-periphery and nested split networks are special cases.

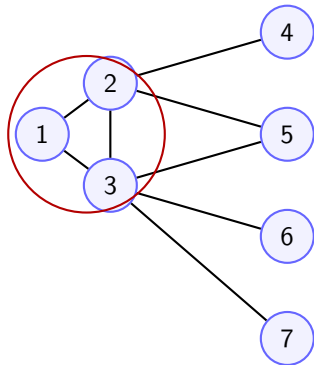


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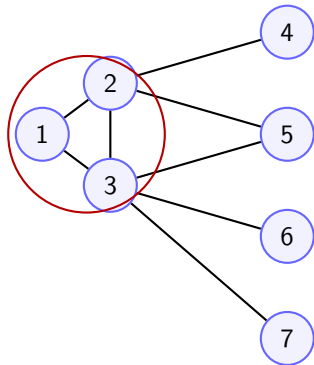


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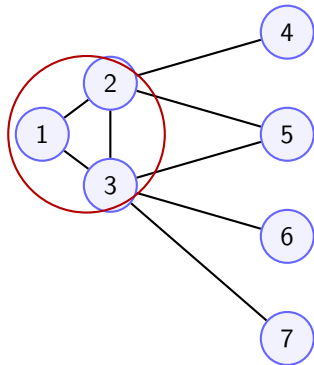


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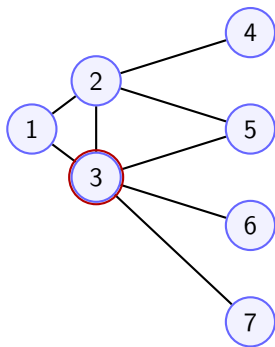
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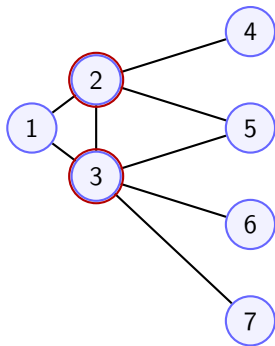
Split Networks: Results

- Medians are coup stable in split networks.
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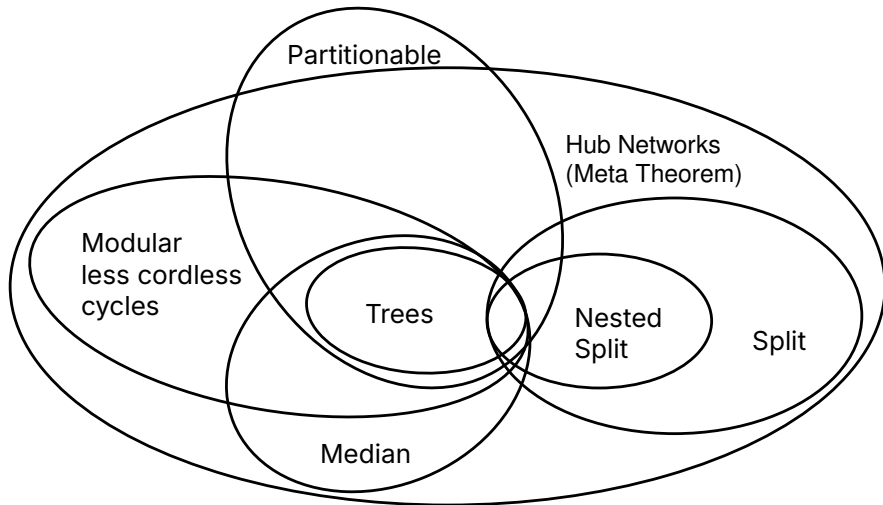


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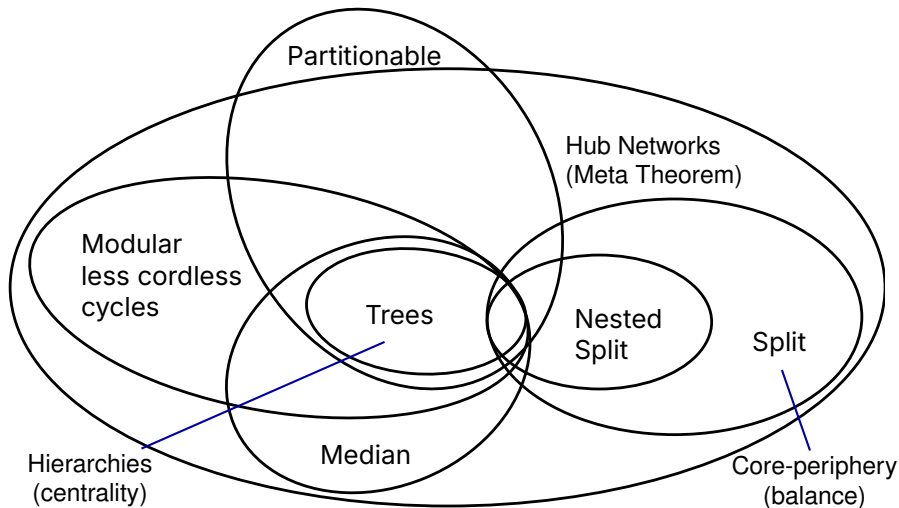
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Which Networks Admit Coup-Stable Nodes



Centrality and Balance



Outline

1 Model

2 Existence

- Centrality: Trees, Median and Modular Networks
- Balance: Core-Periphery Networks

3 Robustness

4 Conclusion

Robustness and Coup-Proofing

- How do coup-stable nodes change when a link is added?
- Why might a government want its military arms siloed?
- Why might a leader want less-connected subordinates?

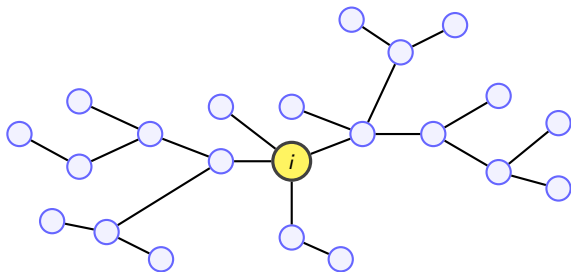
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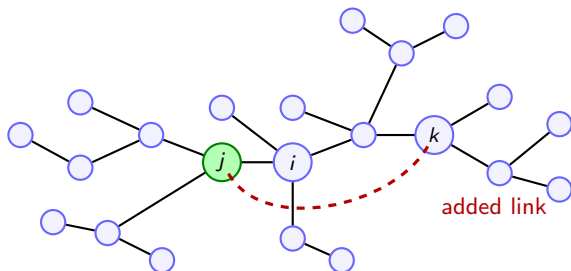
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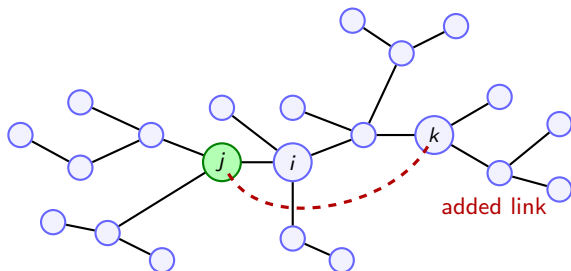
- On a tree, the median i is the unique coup-stable node.
- One lateral link between j and k : the 5 nodes behind k now reach j without passing through i .
- j now has $8 + 5 = 13 > 11$ supporters, defeats i , and becomes the unique coup-stable node.
- Leaders of hierarchies must police ties across branches, and keep branches small.

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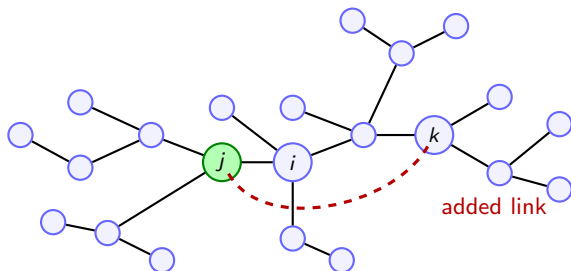
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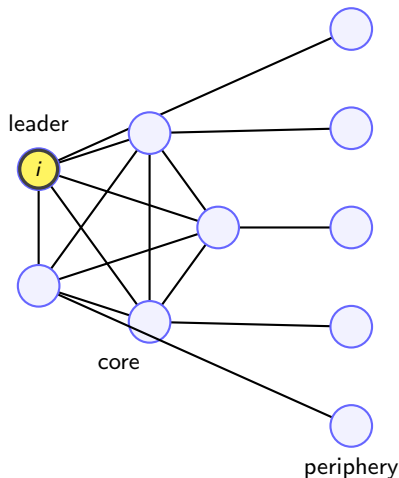
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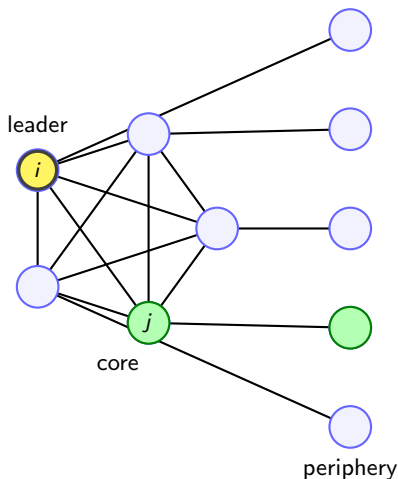
Balance Gives a Buffer

- A core of 5 nodes, each with one follower ($n = 10$). Every core node is coup stable.
- A challenger j gets only itself and its own follower: 2 votes, and it needs 6.
- Each change to a follower's links moves at most one vote: it takes 5 link changes to depose the leader.



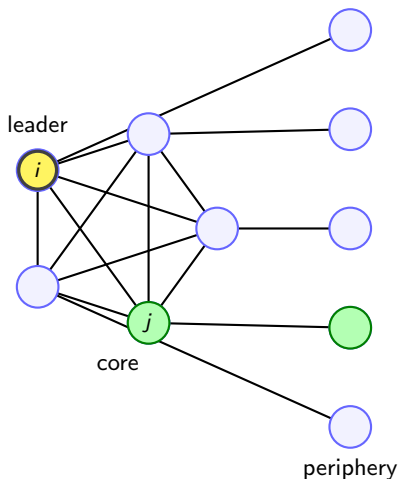
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Conclusion

- Many networks admit coup-stable nodes, and many do not.
- Two logics of coup stability:
 - **centrality**: a highly central node wins against, or does not lose to, all others;
 - **balance**: a core of nodes with many indifferences, none of which defeats another.
- Very different robustness:
 - centrality is fragile: a single link can shift enough votes to change the coup-stable set;
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 - **balance**: a core of nodes with many indifferences, none of which defeats another.
- Very different robustness:
 - centrality is fragile: a single link can shift enough votes to change the coup-stable set;
 - balance gives a buffer: a single link shifts at most one vote.

Conclusion

- Many networks admit coup-stable nodes, and many do not.
- Two logics of coup stability:
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