

Information and Misinformation in Networks

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Overview

Introduction

Existing Literature

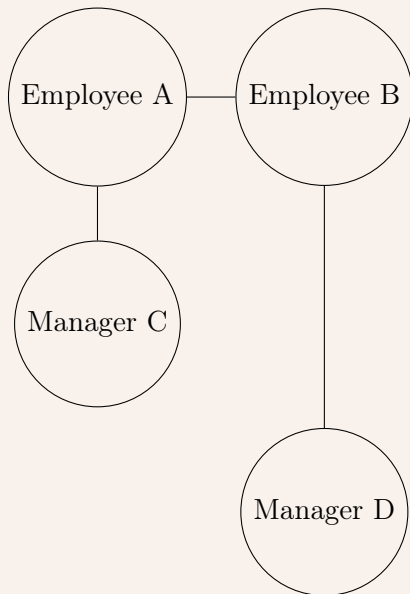
Motivation

The Model

Theorems and Propositions

Conclusion

Introduction



- Employee A shares updates with Employee B and Manager C.
- A tells the truth to C, who works only with A.
- A misleads B, who reports elsewhere and may compete for credit.

Existing Literature

Hagenbach, Koessler, RES, 2010; Galeotti et al., JET, 2013

Truthful reporting happens when the difference between weighted biases of concerned players is less than a threshold value. Weights depend on direct neighbours.

Crawford, Sobel, Econometrica, 1982 (& cheap talk literature)

Misreporting happens due to difference in preferences (captured by **bias**). As the bias tends to 0, communication becomes truthful.

Motivation

- A network is given and players communicate with their neighbours.
- Role of network in communication.
- In case of naive neighbour, when can truthful reporting occur? And when misreporting?
- What if players can choose their bias as well?

The Model: Mathematical Framework

- Set of players: $N \equiv \{1, 2, \dots, n\}$.
- g denotes the graph. $g_{ij} = 1$ if i is connected to j and $g_{ij} = 0$ otherwise. $g_{ii} = 0 \ \forall i$.
- Set of neighbors of i : $N_i \equiv \{j : g_{ij} = 1, j \in N\}$ and $n_i \equiv \sum_{j \in N_i} g_{ij}$.
- $G \equiv [g_{ij}]_{i,j=1}^n$ be the adjacency matrix which represents the network.

Game Timeline

Stage 1: Player i receives a private signal about an uncertain state of the world.

Stage 2: Player i sends and receives messages from $j \in N_i$.

Stage 3: Player i takes action a_i .

Stage 4: Utility is realised.

Interim Stage
(Players maximise expected utility)

Expost Stage

Signal and Message

- Let θ be the state of the world. Each player i receives a private signal $s_i \in [0, 1]$ about θ .
- $\mathbf{s} \equiv (s_i)_{i \in N}$ and $\theta(\mathbf{s}) \equiv \sum_{i \in N} s_i$ (Hagenbach, Koessler (2010)).
- $m_{ij} \in [0, 1]$ be the message sent by player i to player j and $m_i \equiv (m_{ij})_{j \in N_i}$.
- Messages received by i : $m_i^r = (m_{ji})_{j \in N_i}$.

Action and Utility

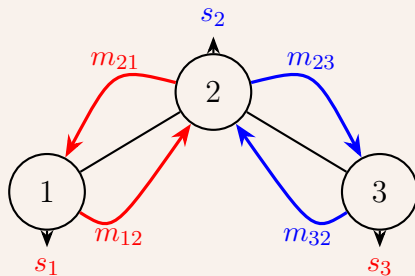
- Action of the player i : $a_i \in A_i = \mathbb{R}$.

$$U_i(a_i, (a_j)_{j \in N_i}, \mathbf{s}) = -\{a_i - (\theta(s) + b_i)\}^2 - \sum_{j \in N_i} (a_i - a_j)^2$$

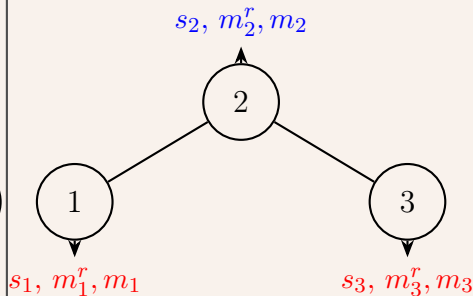
where $b_i \in \mathbb{R}$ is the bias of i .

Equilibrium

[Graphical Representation]



Stage 1: Message Sending



Stage 2: Action Choice

Centrality

- Recall $G = [g_{ij}]_{i,j \in N}$. Now define, $G^N = [\frac{g_{ij}}{1+n_i}]_{i,j \in N}$.
(Ghiglino and Goyal (2010))
- $C \equiv (I - G^N)^{-1} \equiv [c_{ij}]_{i,j=1}^n$.
- $(I - G^N)^{-1}$ exists.

Second Stage Equilibrium (Strategy)

Proposition 1: For any given N and g , the unique Perfect Bayesian equilibrium strategy of player i is given by,

$$\tau_i(m_i, m_i^r, s_i) = \sum_{j \in N} c_{ij} \frac{\theta_j(s) + b_j}{1 + n_j} \equiv \sum_{j \in N} c'_{ij} (\theta_j(s) + b_j).$$

- $c'_{ij} \equiv \frac{c_{ij}}{1 + n_j}$
- $c'_{ij} < c'_{ii}$
- $\sum_{j \in N} c'_{ij} = 1$
- $n_i = 0 \implies \tau_i(m_i, m_i^r, s_i) = \theta_i(s) + b_i.$
- $\lim_{n_i \rightarrow \infty} c'_{ii} = 0.$

Transmission of Message

First we consider **Bayesian Neighbour**.

Later we analyse reporting to a **Naive Neighbour**.

Communication (Bayesian Neighbour)

Theorem 1: In a **complete network**, as

$$(b_1, \dots, b_n) \rightarrow (b, \dots, b),$$

each player can communicate increasingly precise information,
and in the limit communication is fully truthful.

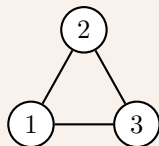
The Role of Network

Let $N = \{1, 2, 3\}$.

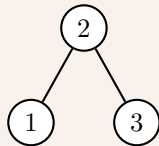
1 Let $(b_1, b_2, b_3) \rightarrow (b, b, b)$.

- Complete network $\Rightarrow$ truthful communication.
- Star network $\Rightarrow$ miscommunication.

Complete network



Star network



2 Let $(b_1, b_2, b_3) = (0, 0.1, 0.3)$.

- Complete network $\Rightarrow$ miscommunication.
- Star network $\Rightarrow$ truthful communication.

Communication (Naive Neighbour)

- For simplicity, we write $\tau_i(m_i, m_i^r, s_i) = \tau_i, \forall i$.
- Suppose, there is a player k who believes player i truthfully reports to him.
- Recall player i sends m_{ik} to k . Let $s_i = \bar{s}_i$.
- When $m_{ik} = \bar{s}_i$, we write $\tau_i = \bar{\tau}_i$ and $\tau_k = \bar{\tau}_k$.

The Modified Utility

$$U_i(a_i, (a_j)_{j \in N_i}, m_i, \mathbf{s}) = -\{a_i - (\theta(s) + b_i)\}^2 - \sum_{j \in N_i} (a_i - a_j)^2$$

$$- \sum_{j \in \mathcal{N}_i} \gamma |m_{ik} - \bar{s}_i|$$

where $\gamma > 0$ be the marginal cost of misreporting and $\mathcal{N}_i \subseteq N_i$:
 $j \in \mathcal{N}_i \iff j$ is naive.

Communication (Bayesian Neighbour)

Let $w_{ik} := \sum_{j \in N_i} c'_{kj}$. Then, $\frac{d\tau_k}{dm_{ik}} = w_{ik}$ and $\frac{d\tau_i}{dm_{ik}} = \frac{w_{ik}}{1+n_i}$.

Proposition 2:

- $m_{ik}^* > \bar{k}_i \iff \bar{\tau}_i - \bar{\tau}_k > \frac{\gamma}{2w_{ik}}.$
- $m_{ik}^* = \bar{k}_i \iff |\bar{\tau}_i - \bar{\tau}_k| \leq \frac{\gamma}{2w_{ik}}.$
- $m_{ik}^* < \bar{k}_i \iff \bar{\tau}_i - \bar{\tau}_k < -\frac{\gamma}{2w_{ik}}.$

Endogenous Bias

- Suppose, players can choose their biases.

How Does Bias Change?

Proposition 2: For any $i, k \in N$,

$$\bullet \quad 0 < \frac{\partial b_i}{\partial b_k} = \frac{\sum_{j \in N_i} c'_{jk}(c'_{ii} - c'_{ji})}{(c'_{ii} - 1)^2 + \sum_{j \in N_i} (c'_{ii} - c'_{ji})^2} < 1.$$
$$\bullet \quad \sum_{k \in N} \frac{\partial b_i}{\partial b_k} = 1.$$

Recall c'_{ij} is the weight player i puts on j 's expected bliss point.
 c'_{ii} is the weight player i puts on his own expected bliss point.

And $0 < c'_{ji} < c'_{ii}$ for any i, j .

Conclusion

- Given a network architecture, how players choose action considering bliss point of other players.
- How misreporting occurs.
- How misreporting depends on centrality.
- How bias changes.

Thank You!

Appendix (Communication Strategy and Second Stage Strategy)

Player i 's **communication strategy** is profile $\beta_i = (\beta_i^j)_{j \in N_i}$ where

$$\beta_i^j : S_i \rightarrow M_{ij}$$

where $\beta_i^j(m_{ij}|s_i)$ is the probability (0 or 1) that player i sends the message m_{ij} to his neighbour j according to his strategy β_i given his private signal realisation is s_i .

Player i 's **second stage strategy** is a mapping defined as:

$$\tau_i : S_i \times M_i \times \mathcal{M}_i \rightarrow A_i.$$

Appendix (Expected State Calculation)

Each player $i \in N$, receives a private signal $\bar{s}_i \in S_i$ about θ , where S_i is a random variable on $[0, 1]$ with distribution function $F_i(\cdot)$ and density function $f_i(\cdot)$.

Expected State of Player i :

$$\begin{aligned}\theta_i^i(\mathbf{s} \mid \bar{s}_i, \mathbf{m}_i) &= \bar{s}_i + \sum_{j \in N_i} \int_{S_j} s_j g_{ji}(s_j \mid m_{ji}) ds_j + \sum_{l \notin N_i \cup \{i\}} \int_{S_l} s_l f_l(s_l) ds_l \\ &\equiv \theta_i(\mathbf{s}).\end{aligned}$$

Expectation formed by i about expected state of $j \in N_i$:

$$\begin{aligned}\nu_j^{i1}(\mathbf{s}) &:= \int_{S_j} s_j g_{ji}(s_j \mid m_{ji}) ds_j. \\ \nu_j^{i2}(\mathbf{s}) &:= \sum_{p \in \tilde{N}_j} \int_{M_{pj}} \int_{S_p} s_p g_{pj}(s_p \mid m_{pj}) ds_p dm_{pj}\end{aligned}$$

where $\tilde{N}_j = N_j \setminus \{(N_j \cap N_i) \cup \{i\}\}$.

Appendix (Expected State Calculation contd...)

$$\nu_j^{i3}(\mathbf{s}) := \sum_{q \in (N_j \cap N_i)} \int_{M_{qj}} \int_{S_q} s_q g_{qj}(s_q \mid m_{qj}) ds_q dm_{qj}.$$

$$\nu_j^{i4}(\mathbf{s}) := \int_{S_i} s_i g_{ij}(s_i \mid m_{ij}) ds_i.$$

$$\nu_j^{i5}(\mathbf{s}) := \sum_{l \notin N_j \cup \{j\}} \int_{S_l} s_l f_l(s_l) ds_l.$$

$$\theta_j^i(\mathbf{s}) := \nu_j^{i1}(\mathbf{s}) + \nu_j^{i2}(\mathbf{s}) + \nu_j^{i3}(\mathbf{s}) + \nu_j^{i4}(\mathbf{s}) + \nu_j^{i5}(\mathbf{s}).$$

Appendix (Defining Equilibrium)

$$(i) \quad \tau^{i,a_i}(\mathbf{s}, (m_j)_{j \in N}) = (a_i, (\tau_j(s_j, m_j, m_j^r))_{j \in N_i}).$$

$$(ii) \quad \tau^i(\mathbf{s}, (m_j)_{j \in N}) = (\tau_i(s_i, m_i, m_i^r), (\tau_j(s_j, m_j, m_j^r))_{j \in N_i}).$$

Appendix (Defining Equilibrium contd...)

An **equilibrium** of the game is simply a strategy profile $(\beta, \tau) = ((\beta_i)_{i \in N}, (\tau_i)_{i \in N})$ satisfying the following properties:

(i) For all $i \in N$, and $s_i \in S_i$,

$$\beta_i(s_i) \in$$

$$\arg \max_{m_i \in M_i} \int_{s_{-i} \in S_{-i}} f_{-i}(s_{-i}) u_i \left(\tau^i(\mathbf{s}, (\beta_j(s_j))_{j \in N}); \mathbf{s}; m_i \right) ds_{-i}.$$

Communication Strategy and Second Stage Strategy

Player i 's **communication strategy** is profile $\beta_i = (\beta_i^j)_{j \in N_i}$ where

$$\beta_i^j : S_i \rightarrow M_{ij}$$

where $\beta_i^j(m_{ij}|s_i)$ is the probability (0 or 1) that player i sends the message m_{ij} to his neighbour j according to his strategy β_i given his private signal realisation is s_i .

Player i 's **second stage strategy** is a mapping defined as:

$$\tau_i : S_i \times M_i \times \mathcal{M}_i \rightarrow A_i.$$

Equilibrium

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Equilibrium (contd...)

(ii) For all $i \in N$, $m_i \in M_i$, and $m_i^r \in \mathcal{M}_i^{\beta-i}$,

$$\tau_i(s_i, m_i, m_i^r)$$

$$\in \arg \max_{a_i \in A_i} \int_{s_{-i} \in S_{-i}} g_i(s_{-i} | m_i^r) u_i(\tau^{i, a_i}(\mathbf{s}, (m_j)_{j \in N}); \mathbf{s}; m_i) ds_{-i},$$

where

$$g_i(s_{-i} | m_i^r) = \prod_{j \in N_i} \frac{\beta_j^i(m_{ji} | s_j) f_j(s_j)}{\int_{t_j \in S_j} \beta_j^i(m_{ji} | t_j) f_j(t_j) dt_j} \prod_{l \notin N_i} f_l(s_l), \text{ and}$$

$\mathcal{M}_i^{\beta-i}$ is the set of messages received by player i given his or her neighbours adapts the equilibrium messaging rule $(\beta_j^i(\cdot))_{j \in N_i}$.

Appendix (Changing a Network...)

Suppose, each player i knows $S_j \sim U[0, 1] \ \forall i$. Everything explained so far except s'_i 's is common knowledge. Suppose this is also a common knowledge that if i sends m_{ij} to j , then j thinks $(S_i \mid m_{ij} \in M_{ij}) \sim U[0, m_{ij}]$. Assume that, player i thinks whatever player 1 is telling to i is truthful and this is common knowledge between i and 1 only, and $i = \{2, 3\}$.

Appendix (Why bias can be a choice variable)

- Suppose staying in Kolkata, I have food and fashion preference aligned with Bengali Culture.
- If I move to USA, over years I may change fashion taste or food preference.
- We try to analyse how should this change?